Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Friday 17 May 2024

Afternoon

Paper reference **8FM0/21**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
21: Further Pure Mathematics 1
(Part of options A, B, C and D)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

1. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(a) Sketch the graph of the curve with equation

$$y = \frac{1}{x^2}$$

(2)

(b) Solve, using algebra, the inequality

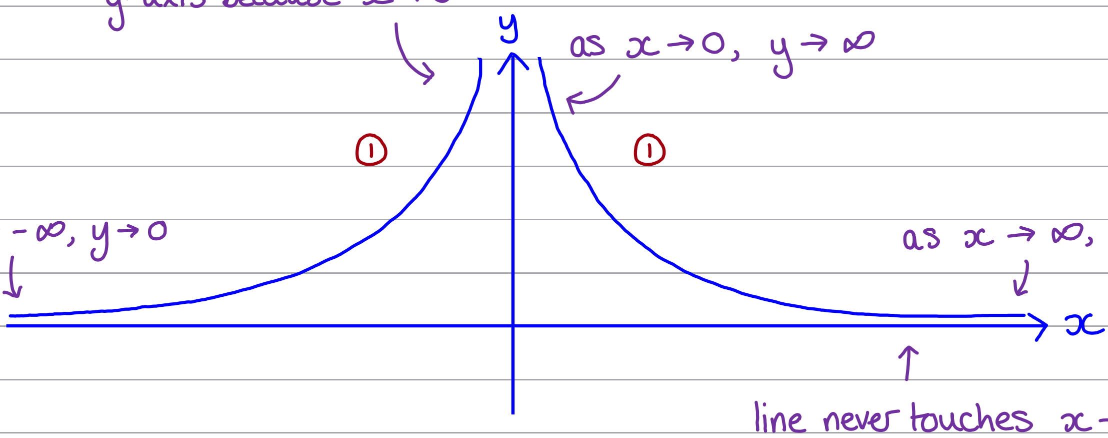
$$3 - 2x^2 > \frac{1}{x^2}$$

(5)

(a)

line never touches y-axis because $x \neq 0$

as $x \to 0$, $y \to \infty$



as $x \to -\infty$, $y \to 0$

as $x \to \infty$, $y \to 0$

line never touches x-axis because $\frac{1}{a} > 0$, $a \in \mathbb{R}$

(b) $3 - 2x^2 > \frac{1}{x^2}$ $\quad \times x^2$

$3x^2 - 2x^4 > 1$

$3x^2 - 2x^4 - 1 > 0$ 1 $\leftarrow (-2x^2+1)(x^2-1) = 0$

$x^2 = 1 \implies x = 1, \; x = -1$ 1

$x^2 = \frac{1}{2} \implies x = -\frac{\sqrt{2}}{2}, \; +\frac{\sqrt{2}}{2}$ 1 $\leftarrow \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

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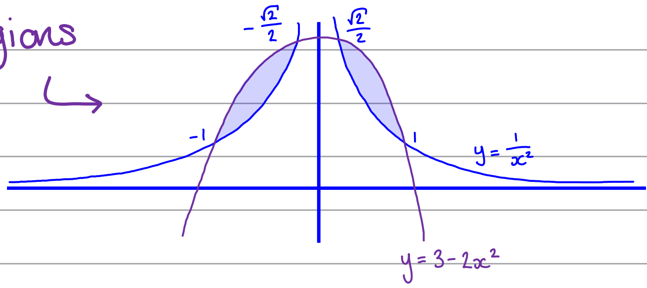
Question 1 continued

Sketch graph to determine shaded regions



$3 - 2x^2 > \frac{1}{x^2}$ when:

$-1 < x < -\frac{\sqrt{2}}{2}$ 1

$\frac{\sqrt{2}}{2} < x < 1$ 1

(Total for Question 1 is 7 marks)

2. An area of woodland contains a mixture of blue and yellow flowers.

A study found that the proportion, x, of blue flowers in the woodland area satisfies the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{xt(0.8 - x)}{x^2 + 5t} \qquad t > 0$$

where t is the number of years since the start of the study.

Given that exactly 3 years after the start of the study half of the flowers in the woodland area were blue,

(a) use one application of the approximation formula $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_n \approx \frac{y_{n+1} - y_n}{h}$ to estimate the proportion of blue flowers in the woodland area half a year later.

(5)

(b) Deduce from the differential equation the proportion of flowers that will be blue in the long term.

(1)

(a) when $t = 3$, $x_0 = \frac{1}{2}$. 1

Step size $h = \frac{1}{2}$ $\leftarrow$ half a year in one step : $h = \frac{1}{2} \div 1$

$$\left(\frac{dx}{dt}\right)_0 = \frac{xt(0.8 - x)}{x^2 + 5t}$$

$$= \frac{\frac{1}{2} \times 3(0.8 - \frac{1}{2})}{(\frac{1}{2})^2 + 5(3)} \quad 1$$

$$= \frac{9}{305}$$

When $t = 3.5$:

x when $t = 3.5$

$$\underbrace{\frac{9}{305}}_{\left(\frac{dx}{dt}\right)_0} \approx \frac{x_{n+1} - \frac{1}{2}}{\underbrace{\frac{1}{2}}_{h}} \quad 1$$

$\leftarrow x$ when $t = 3$

$$\frac{1}{2} \times \frac{9}{305} + \frac{1}{2} = \frac{157}{305}$$

$x_2 \approx 0.515$ (3.s.f) 1

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Question 2 continued

(b) 80% blue flowers in the long term

As t increases, the denominator $5x^2 + 5t$ becomes very large,
so $\frac{dx}{dt}$ tends toward 0. For $\frac{dx}{dt} = 0$, $0.8 - x = 0$ $\therefore x = 0.8$

(Total for Question 2 is 6 marks)

3. Vectors **u** and **v** are given by

$$\mathbf{u} = 5\mathbf{i} + 4\mathbf{j} - 3\mathbf{k} \quad \text{and} \quad \mathbf{v} = a\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}$$

where a is a constant.

(a) Determine, in terms of a, the vector product $\mathbf{u} \times \mathbf{v}$

(2)

Given that

- $\overrightarrow{AB} = 2\mathbf{u}$
- $\overrightarrow{AC} = \mathbf{v}$
- the area of triangle ABC is 15

(b) determine the possible values of a.

(4)

(a) $u \times v = \begin{array}{c} \\ u \rightarrow \\ v \rightarrow \end{array} \begin{vmatrix} i & j & k \\ 5 & 4 & -3 \\ a & -6 & 2 \end{vmatrix}$ ← use 3×3 determinant to find the vector product (1)

$= (4\times 2 - -3\times -6)i - (5\times 2 - -3\times a)j + (5\times -6 - 4\times a)k$

$= -10i - (10+3a)j - (30+4a)k$ (1)

(b) Area $= \frac{1}{2}|2u \times v| = 15$

$\frac{1}{2} \times 2|-10i - (10+3a)j - (30+4a)k| = 15$ (1)

$2u\times v$

$\sqrt{(-10)^2 + (10+3a)^2 + (30+4a)^2} = 15$

$100 + 100 + 60a + 9a^2 + 900 + 240a + 16a^2 = 15^2$

$25a^2 + 300a + 875 = 0$ (1)

$\div 25$

$a^2 + 12a + 35 = 0$

$(a+7)(a+5) = 0$ (1) $\Longrightarrow a = -7$ or -5 (1)

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Question 3 continued

(Total for Question 3 is 6 marks)

4. (a) Given that $t = \tan \frac{x}{2}$ prove that

$$\cos x \equiv \frac{1 - t^2}{1 + t^2}$$

(3)

(b) Show that the equation

$$3\tan x - 10\cos x = 10$$

can be written in the form

$$(t + 2)\left(at^2 + bt + c\right) = 0$$

where $t = \tan \frac{x}{2}$ and a, b and c are integers to be determined.

(4)

(c) Hence solve, for $-180° < x < 180°$, the equation

$$3\tan x - 10\cos x = 10$$

(5)

(a) $\cos 2x = \cos^2 x - \sin^2 x \implies \cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}$ 1

Using t-formulae identities

$\cos x = \left(\frac{1}{\sqrt{1+t^2}}\right)^2 - \left(\frac{t}{\sqrt{1+t^2}}\right)^2$ 1

$\cos x = \frac{1}{\sqrt{1+t^2}}$

$\sin x = \frac{t}{\sqrt{1+t^2}}$

$\cos x = \frac{1}{1+t^2} - \frac{t^2}{1+t^2}$ 1

$\therefore \cos x \equiv \frac{1-t^2}{1+t^2}$

(b) $3\tan x - 10\cos x = 10$

$3 \times 2\left(\frac{1}{1-t^2}\right) - 10 \times \left(\frac{1-t^2}{1+t^2}\right) = 10$ 1

$\frac{6}{1-t^2} - \frac{10(1-t^2)}{1+t^2} = 10$

$6(1+t^2) - 10(1-t^2)(1-t^2) = 10(1+t^2)(1-t^2)$

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Question 4 continued

$$6t + 6t^3 - 10 + 20t^2 - 10t^4 = 10t - 10t^4$$

$$6t^3 + 20t^2 + 6t - 20 = 0 \quad 1$$

$$\begin{array}{r} 3t^2 + 4t - 5 \\ t+2 \overline{)\, 3t^3 + 10t^2 + 3t - 10} \\ \underline{3t^3 + 6t^2} \\ \underline{4t^2 + 8t} \\ -5t - 10 \end{array}$$

$$3t^3 + 10t^2 + 3t - 10 = 0$$

$$(t+2)(3t^2 + 4t - 5) = 0 \quad 2$$

(c) $t + 2 = 0 \Rightarrow \tan\frac{x}{2} + 2 = 0$

$$\tan\frac{x}{2} = -2$$

$$\frac{x}{2} = \tan^{-1}(-2)$$

$$x = 2\tan^{-1}(-2) \quad 1$$

$$x = 2 \times -63\ 43$$

only value in range $-180 < x < 180$ $\longrightarrow$ $x = -126.87° \quad 1$

$$3t^2 + 4t - 5 = 0 \Rightarrow t = \frac{-4 \pm \sqrt{16 - 4 \times 3 \times -5}}{6}$$

$$\tan\frac{x}{2} = \frac{-2 \pm \sqrt{19}}{3}$$

$$x = 2 \times \tan^{-1}\left(\frac{-2 \pm \sqrt{19}}{3}\right) \quad 1$$

only values in range $-180 < x < 180$ $\longrightarrow$ $x = 76.36°\ 1,\ -129.49°\ 1$

Question 4 continued

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Question 4 continued

(Total for Question 4 is 12 marks)

5. The parabola C has equation $y^2 = 16x$

The point P on C has y coordinate p, where p is a positive constant.

(a) Show that an equation of the tangent to C at P is given by

$$2py = 16x + p^2$$

[You may quote without proof that for the general parabola $y^2 = 4ax$, $\frac{dy}{dx} = \frac{2a}{y}$]

(2)

(b) Write down the equation of the directrix of C.

(1)

The line l is the reflection of the tangent to C at P in the directrix of C.

Given that l passes through the focus of C,

(c) determine the exact value of p.

(6)

(a) At point P. $y = p$ $\quad p^2 = 16x \Rightarrow \frac{p^2}{16} = x$

$y^2 = 16x \Rightarrow y = \sqrt{16}\sqrt{x} = 4x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2} \times 4 \times x^{\frac{1}{2}-1}$

$\frac{dy}{dx} = \frac{2}{\sqrt{x}} = 2 \times \frac{1}{\sqrt{\frac{p^2}{16}}} = \frac{8}{p}$

Tangent at P. $y = \frac{8}{p}x + c \quad \leftarrow \quad y = mx + c$

$p = \frac{8}{p} \times \frac{p^2}{16} + c \quad \leftarrow$ when $y = p$, $x = \frac{p^2}{16}$

$p - \frac{8p^2}{16p} = c \Longrightarrow \frac{p}{2} = c$ 1

$y = \frac{8}{p}x + \frac{p}{2}$ 1 $\times 2$

$2y = \frac{16}{p}x + p$ $\times p$

$2py = 16x + p^2$

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Question 5 continued

(b) $x = -4$ 1 $\leftarrow$ $y^2 = 4ax$ and directrix $x + a = 0$.

(c) $y^2 = 4(4)x$ $\therefore$ focus of C is at $(4, 0)$

gradient of L = reflection of m_p = $-\frac{8}{p}$ 1

Tangent meets directrix when $2py = 16x + p^2$ 1

$$2py = 16(-4) + p^2$$

$$y_1 = \frac{p^2 - 64}{2p}$$ 1

Equation of L : $y - \frac{p^2-64}{2p} = -\frac{8}{p}(x-(-4))$ 1 $\leftarrow$ $y - y_1 = m(x - x_1)$

Focus at $y=0$, $x = 4$: $0 - \frac{p^2-64}{2p} = -\frac{8}{p}(4+4)$ 1

$$\frac{p^2-64}{2p} = -\frac{64}{p}$$ $\times -2p$

$$p^2 - 64 = 128$$

$$p^2 = 192$$

$p = 8\sqrt{3}$ 1 $\leftarrow$ $p \geq 0$

Question 5 continued

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Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

TOTAL FOR FURTHER PURE MATHEMATICS 1 IS 40 MARKS